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Josephson current oscillation in a Rashba ring

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Abstract
We propose to use Rashba spin–orbital coupling (RSOC) to modulate the Josephson current in a one-dimensional ring-SQUID (superconducting quantum interference device) without any magnetic field or magnetic material. Supercurrent oscillation due to spin-dependent quantum interference of spin wavefunctions passing through the two arms of the ring in the presence of RSOC is found. A pure spin current flows in the system's ground state without a supercurrent due to the combined effect of the RSOC precession phase and the electron traveling phase caused by the asymmetry in the two arms of the ring. This purely electrical means of modulating the supercurrent by varying the RSOC will be useful for designing novel superconductor devices.

Recently, the Josephson current modulation in a superconductor/ferromagnet/superconductor (SC/FM/SC) heterojunction has attracted considerable attention due to its experimental observation and potential applications in spintronics and quantum computing [1–3]. The physical origin of the Josephson effect is the macroscopic phase difference of two SCs and this can be understood through the Andreev reflection processes of quasiparticles with energy smaller than the superconducting energy gap. An electron impinging on one of the interfaces is Andreev reflected and converted into a hole moving in the opposite direction, thus generating a Cooper pair in a SC; this hole is consequently Andreev reflected at the second interface and is converted back to an electron, leading to the destruction of a Cooper pair in the other SC. As a result of this cycle, a pair of correlated electrons is transferred from one SC to another, creating a supercurrent flow across the junction.

The Josephson current is known to be a periodic function of the macroscopic phase difference ϕ of two SCs in the junction, i.e., $I \sim I_c \sin\phi$ with I_c being the superconductor critical current. It has been shown experimentally and theoretically that the I_c direction can be reversed in some cases due to an additional π phase shift, i.e., resulting in a 0–π transition, $I \sim I_c \sin(\phi + \pi)$; then the Josephson junction is called a π junction or said to be in a π state. The 0–π transition is mainly observed in SC/FM/SC Josephson junctions—the FM not only destroys the conventional superconductivity correlation through the FM exchange field, but also causes Cooper pairs to have nonzero momentum; as a result, the Josephson current exhibits a damped oscillation with the FM layer length or FM exchange strength [4–9]. Due to the difficulty in tuning the FM layer length in a single experiment, some alternatives have been proposed for observing the 0–π transition of the Josephson current. For example, the SC/FM/FM/SC junction with noncollinear magnetizations in the two FMs has been investigated by Pajović *et al* [10] and the π state can be found by varying the relative direction of the FM moments.

The Rashba spin–orbital coupling (RSOC) has also been suggested for modulating the supercurrent, not only because the RSOC can lead to a spin splitting of the electron energy band, but also, and more importantly, because RSOC in a semiconductor is easy to integrate into devices and makes purely electrical control of devices possible without using any FM element or magnetic field. A few studies [11–14] have shown that in one-dimensional cases the RSOC can hardly exert any effect on the supercurrent unless the Zeeman splitting resulting from a magnetic field is included, since the RSOC does not destroy the time-reversal symmetry. However, Dell'Anna *et al* [14] argued that the time-reversal symmetry has already been broken by the supercurrent, and found that the RSOC can have a huge effect on the Josephson current in a quantum dot system. Very recently, Reynoso *et al* [15] have found that the RSOC can significantly modulate the supercurrent in a quantum point contact device with the aid of an in-plane magnetic field. In this work, we propose to use a 1D Aharonov–Bohm (AB) ring with RSOC [16–20] to modulate the Josephson current. Although the spin-up and spin-down electron precession due to RSOC cannot lead to an extra phase in a Cooper pair, in a ring structure the Josephson current can indeed exhibit an oscillation due to the quantum interference of spin-resolved wavefunctions traveling in the upper and lower arms of the ring. It is also found that when the system is in the ground state with zero SC macroscopic phase difference between the two SC leads, a pure spin current is flowing through the device because of the combined effect of the asymmetry in the ring's arms and spin precession phases.

We consider a ring-SQUID structure schematically shown in figure 1 where two SC leads are asymmetrically connected with a ring with RSOC, i.e., the lengths of the upper arm (L_1) and lower arm (L_2) can be different. The Hamiltonian of the ring with RSOC is given by

$$H = \frac{\mathbf{p}^2}{2m} + \frac{\alpha}{\hbar}(p_y\sigma_x - p_x\sigma_y) \tag{1}$$

where the first term is the free electron one and the second is the RSOC term with $\mathbf{p}$ denoting the electron momentum, $\vec{\sigma}$ the Pauli matrix, and m the electron effective mass. The RSOC strength α can be varied by means of an electric field [21]. Within the second-quantization formalism [20], the total Hamiltonian of the system can be given by

$$H = H_{\rm L} + H_{\rm R} + H_{\rm rsoc} + H_T, \tag{2}$$

$$H_{i(={\rm L,R})} = \sum_{ki\sigma} \varepsilon_{ki\sigma} C^\dagger_{ki\sigma} C_{ki\sigma} + \sum_{ki\sigma}(\Delta_i C^\dagger_{k\uparrow} C^\dagger_{-k\downarrow} + \text{c.c.}), \tag{3}$$

$$H_{\rm rsoc} = H_1 + H_2 = \sum_{n\sigma,n=1,2} \varepsilon_n d^\dagger_{n\sigma} d_{n\sigma}, \tag{4}$$

$$H_T = \sum_{ki\sigma}(T_{ki\sigma,n\sigma} C^\dagger_{ki\sigma} d_{n\sigma} + \text{c.c}), \tag{5}$$

where $C^\dagger_{ki\sigma}$ ($C_{ki\sigma}$) and $d^\dagger_{n\sigma}$ ($d_{n\sigma}$) are the creation (annihilation) operators of the SC leads and the Rashba ring with spin $\sigma = \uparrow\downarrow = \pm$, respectively; H_i describes the left and right SC leads; $H_{\rm rsoc}$ stands for the upper ($n = 1$) and lower ($n = 2$) branches of the ring where the site energies (ε_1, ε_2) are set different for generality; H_T is the coupling between the SC leads and the ring. Without loss of generality, the right SC macroscopic phase is set to zero and the left one ϕ is transformed into the left component of the tunneling Hamiltonian given as

$$T_{k{\rm L}\sigma,n\sigma} = t_{k{\rm L},n} {\rm e}^{{\rm i}\phi/2}, \tag{6}$$

where $t_{k{\rm L},n}$ is a spin-independent hopping matrix. The spin precession phase from the RSOC when electrons travel through the ring is given by

$$T_{k{\rm R}\sigma,n\sigma} = t_{k{\rm L},n} {\rm e}^{{\rm i}\sigma\varphi_n}, \tag{7}$$

where $\varphi_n = \alpha m L_n/\hbar^2$ is also referred to as the Aharonov–Casher (AC) dynamic phase; we neglect the chirality of the spin precession phase φ here when the electron moves in the upper arm and lower arm of the ring, i.e., φ_1 and φ_2 have a sign difference in a uniform ring, and therefore, our model

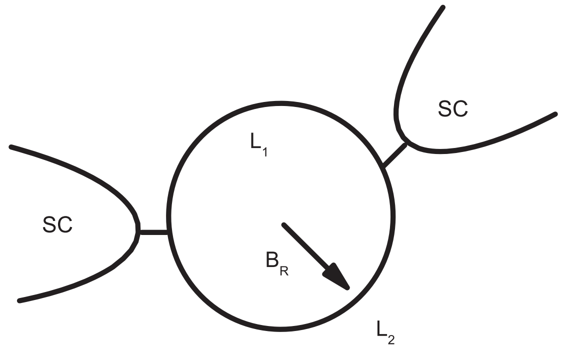

Figure 1. The schematic of a Rashba ring-SQUID in the xy plane, in which the two SC leads are asymmetrically connected with the ring. The lengths of the upper and lower arms of the ring are L_1 and L_2 respectively. $B_{\rm R}$ is the pseudomagnetic field from the RSOC, which is along the radial direction.

is also suitable for describing the hybrid ring. The up-spins and down-spins have acquired exactly opposite phases from the RSOC so generally, a Cooper pair of the SC composed of up-spin and down-spin electrons is not affected by the RSOC. The asymmetry of the ring $\varepsilon_1 \neq \varepsilon_2$ is crucial for the quantum interference effect in our model and it can be introduced by placing a scatterer in one arm of the ring or by locally applying a gate voltage that affects the properties of one arm [22]. Here $\varepsilon_1 \neq \varepsilon_2$ can exert a similar effect to the propagating phase difference of electrons traveling in the upper and lower arms of a ring.

We focus on the supercurrent through the ring. The spin-dependent current density operator is given by $\hat{I}_{{\rm L}\sigma} = e\frac{{\rm d}\hat{N}_{{\rm L}\sigma}}{{\rm d}t} = \frac{e}{{\rm i}\hbar}[\hat{N}_{{\rm L}\sigma}, H_T]$ where $\hat{N}_{{\rm L}\sigma}$ is the σ-spin electron operator in the left SC, and after commutation the steady current density reads

$$I_{{\rm L}\sigma} = \frac{2e}{\hbar} {\rm Re} \int \frac{{\rm d}\omega}{2\pi} \sum_{kn} T_{k{\rm L}\sigma,n\sigma} G^<_{n\sigma,k{\rm L}\sigma}(\omega). \tag{8}$$

$G^<_{n\sigma,k{\rm L}\sigma'}(t,t') = {\rm i}\langle C^\dagger_{k{\rm L}\sigma'}(t') d_{n\sigma}(t)\rangle$ is the lesser Green's function, where $\langle\cdots\rangle$ denotes the quantum statistical average. Since we focus on the equilibrium case where zero bias is applied on the system, the lesser Green's function can be decomposed using the Keldysh equation as

$$G^<(\omega) = [G^{\rm a}(\omega) - G^{\rm r}(\omega)] f(\omega), \tag{9}$$

where $G^{{\rm a}\,({\rm r})}(\omega)$ is the usual advanced (retarded) Green's function and $f(\omega)$ is the Fermi–Dirac distribution function. Using the Dyson equation, the equation (8) can be simplified as

$$I_{{\rm L}\sigma} = \frac{e}{2\hbar} \int \frac{{\rm d}\omega}{2\pi} f(\omega) \,{\rm Tr}\{[G^{\rm a}_{d\sigma} \Xi^{\rm a}_{{\rm L}\sigma} - G^{\rm r}_{d\sigma} \Xi^{\rm r}_{{\rm L}\sigma} + \Xi^{\rm r}_{{\rm L}\sigma} G^{\rm r}_{d\sigma} - \Xi^{\rm a}_{{\rm L}\sigma} G^{\rm a}_{d\sigma}]\tau\}, \tag{10}$$

where $G^{\rm r,a}_d$ is the Green's function of the ring coupled with two SC leads and it is given by direct matrix inversion $G^{\rm r,a}_{d\sigma} = [\omega I - \tilde{H}_{\rm rsoc} - \Xi^{\rm r,a}_{{\rm L}\sigma} - \Xi^{\rm r,a}_{{\rm R}\sigma}]$, $\tilde{H}_{\rm rsoc}$ is equation (4) in Nambu space; $\Xi^{\rm r,a}_{{\rm L\,(R)}\sigma} = \sum_{k{\rm L\,(R)},nm} T_{k{\rm L}\sigma,n\sigma} g^{\rm r,a}_{k{\rm L\,(R)}\sigma} T_{m\sigma,k{\rm L\,(R)}\sigma}$ is the self-energy from the left (right) SC lead with $g^{\rm r,a}_{k{\rm L\,(R)}\sigma}$ the uncoupled Green's function of the two SC leads and is spin independent. $\tau = \sigma_0 \otimes \sigma_z$ is a 4×4 matrix with σ_z the Pauli matrix and σ_0 the unit matrix.

For simplicity, the left and right SC pair potentials are assumed equal, $\Delta_L = \Delta_R = \Delta$, and the hopping matrix elements are identical too, $|t_{kL,n}|^2 = |t_{kR,n}|^2$. From direct algebra, the spin-resolved current is given by

$$I_{L\sigma} = \frac{2e}{\hbar} \int \frac{d\omega\, \Gamma^2 \Delta^2}{2\pi(\omega^2 - \Delta^2)} [(\varepsilon_1 \varepsilon_2 - \omega^2)(1 + \cos\varphi)\sin\phi + \tfrac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 \sin\phi + (\varepsilon_1 - \varepsilon_2)\omega\cos\phi\sin\sigma\varphi] \times f(\omega)/\,\mathrm{Im}(Y), \tag{11}$$

$$Y = \mathrm{Det}[g_1^{-1}]\mathrm{Det}[g_1^{-1} - (\Sigma_L + \Sigma_R e^{-i\sigma\varphi}) g_2 (\Sigma_L + \Sigma_R e^{i\sigma\varphi})], \tag{12}$$

where $\varphi = \varphi_1 - \varphi_2$ is the spin precession phase difference between the two arms of the ring and $\Gamma = 2\pi |t_{ki,n}|^2 \rho_N$ is the linewidth function, which is assumed independent of energy ω, with ρ_N being the normal density of states of the SC lead. $g_{1(2)} = [\omega I - H_{1(2)} - \Sigma_L - \Sigma_R]^{-1}$ is the retarded Green's function of the upper (lower) arm of the ring with $H_{1(2)}$ of equation (4) expressed in the Nambu space and Σ_L is the retarded self-energy from the left SC lead,

$$\Sigma_L = \frac{-i\beta(\omega)\Gamma}{2} \begin{pmatrix} 1 & -\Delta e^{i\phi}/\omega \\ -\Delta e^{-i\phi}/\omega & 1 \end{pmatrix}, \tag{13}$$

$\beta(\omega)$ is defined as $\beta(\omega) = |\omega|/\sqrt{\omega^2 - \Delta^2}$ for $|\omega| > \Delta$ and $\beta(\omega) = \omega/i\sqrt{\Delta^2 - \omega^2}$ for $|\omega| < \Delta$. The right self-energy Σ_R is set equal to the left one by putting $\phi = 0$ in equation (13).

In a *uniform* ring, $\varepsilon_1 = \varepsilon_2$, the second and third terms in equation (11) vanish and the first term remains nonzero, so the spin precession phase from RSOC can still affect the supercurrent $I_L = I_{L\uparrow} + I_{L\downarrow} \sim (1 + \cos\varphi)$ $(I_{L\uparrow} = I_{L\downarrow})$, even though a Cooper pair itself cannot acquire any additional phase from the RSOC, unlike traveling in the FM. In other words, the modulation of the supercurrent in equation (11) stems actually from the quantum interference of spin wavefunctions traveling through the two arms, which is same as the conductance of the Rashba ring [16–19]. In figure 2, the supercurrent I_L through a uniform ring is plotted as a function of the precession phase difference $\varphi = \alpha m(L_1 - L_2)/\hbar^2$ with two different linewidth functions Γ, and exhibits a sinusoidal oscillation at both strong and weak coupling. At destructive interference $\varphi = (2m+1)\pi$ (m integer), I_L vanishes. Hence the nonzero phase difference φ is important for the quantum interference. As is well known, the RSOC constant can be varied by means of a perpendicular electric field. By varying the RSOC [21] electrically, the Josephson current in our structure can be correspondingly modulated without using any magnetic element. It is pointed out here that the 0–π transition of Josephson current is still absent in this 1D Rashba ring-SQUID structure.

According to the spin-dependent current $I_{L\sigma}$ expression (equation (11)), a pure spin current without charge current can flow through the ring when $\phi = 0$ and $\varepsilon_1 \neq \varepsilon_2$, $I_\uparrow = -I_\uparrow$. This originates from the combined effect of the spin precession phase difference φ due to RSOC and the usual electron wavefunction phase difference. The latter will occur when the site energies ε_1 and ε_2 are different and an electron acquires different phases by traveling through the two arms of

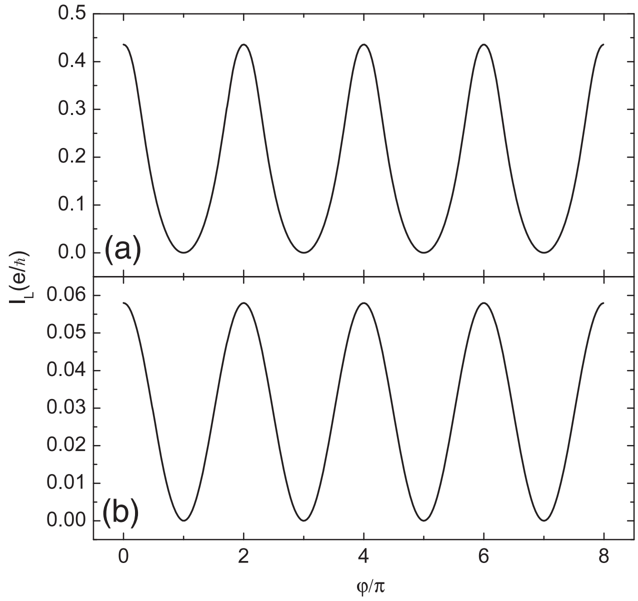

Figure 2. The Josephson current I_L shown as a function of the spin precession phase difference φ for a uniform ring with $\varepsilon_1 = \varepsilon_2 = 0$, the order parameter $\Delta = 1$, temperature $T = 0$ K, left SC macroscopic phase $\phi = \pi/2$, $\Gamma = 0.5\Delta$ in (a), and $\Gamma = 0.05\Delta$ in (b).

the ring. Actually, the third term in equation (11) accounts for the spontaneous spin polarization in a nonuniform RSOC system [23]. Assuming the electron transmission amplitudes in the upper arm t_1 and the lower arm t_2 to have a phase difference, $t_1 = t_2 e^{i\theta}$, due to the asymmetry $\varepsilon_1 \neq \varepsilon_2$ ($\theta = 0$ when $\varepsilon_1 = \varepsilon_2$), the total spin-dependent transmission is $T_\sigma = |t_1 + t_2 e^{i\sigma\varphi}|^2$ so the spontaneous spin polarization $T_\uparrow - T_\downarrow \sim \sin\theta \sin\varphi$ can form. Only in the currently studied ring-SQUID can a pure spin current $J_z = I_{L\uparrow} - I_{L\downarrow}$ flow in the ground state of the system when $\phi = 0$, and it is absent in a normal RSOC system. In figure 3, spin current $J_z = I_{L\uparrow} - I_{L\downarrow}$ is plotted as a function of φ with different Γ values and exhibits a non-sinusoidal oscillating behavior. The magnitude of the pure spin current is much smaller than that of the supercurrent since in this case, $\phi = 0$, the Andreev bound state cannot contribute to the current in the SC energy gap. The pure spin current exhibits the '0–π' transition and its direction can be reversed by modulating either the spin precession phase difference φ or the arm asymmetry $\varepsilon_1 - \varepsilon_2$. As a consequence, in the general case $\phi \neq 0$ and $\varepsilon_1 \neq \varepsilon_2$, the current is spin polarized, $I_{L\uparrow} \neq I_{L\downarrow}$, in this SC Rashba ring, as in the nonuniform normal Rashba system studied in [23].

So far we have considered a 1D Rashba ring and a single transport channel. It is noted that in a finite-size ring more modes are involved in transport and the Josephson current modulation by the RSOC in our model can still work, although the interband (mode) coupling can smear the interference effect. Many authors have demonstrated that in multi-mode Rashba rings, the conductance can also oscillate with the RSOC constant, as for the one-mode ring. The reason is that the phase difference due to spin precession in the two arms is independent of the wavevector or electron energy [16, 24]

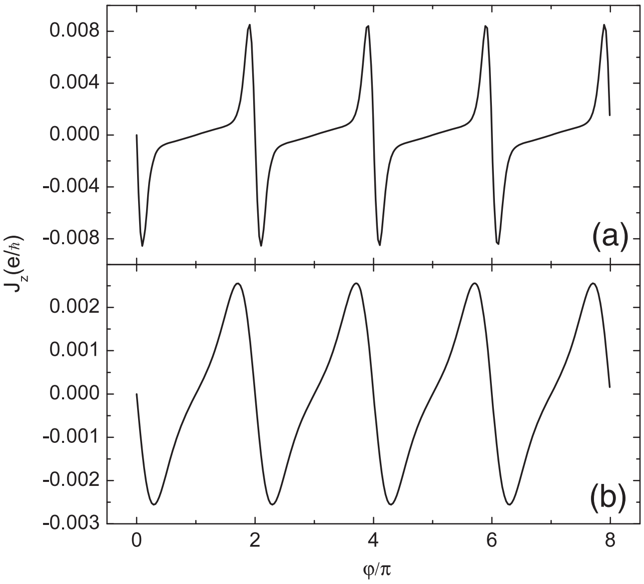

Figure 3. The pure spin current J_z as a function of the spin precession phase difference φ, with $\Delta = 1$, $T = 0$ K, $\phi = 0$, $\varepsilon_1 = -\varepsilon_2 = 0.5\Delta$, $\Gamma = 0.5\Delta$ in (a), and $\Gamma = 0.05\Delta$ in (b).

of the modes in the upper and lower arms. For a uniform RSOC ring [16], the AC phase φ is estimated to be 7.4π for a 0.3 μm ring radius with the RSOC constant $\alpha \sim 1.05 \times 10^{-11}$ eV m, which is sufficient for modulating the oscillation of the supercurrent as well as the pure spin current.

In summary, we proposed a 1D Rashba ring-SQUID structure for modulating the Josephson current. Due to quantum interference, the spin precession due to the RSOC in the two arms of the ring will result in an oscillating behavior of the supercurrent, although the Cooper pair cannot acquire an additional phase from the RSOC. When there is no SC phase difference between the two SC leads, a pure spin current can flow through the ring due to the combined effect of the spin precession phase and the spatial wavefunction phase. The oscillation of the supercurrent in our proposal can be realized using purely electric means without any magnetic factor, which will be useful in the design of novel superconductor devices.

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